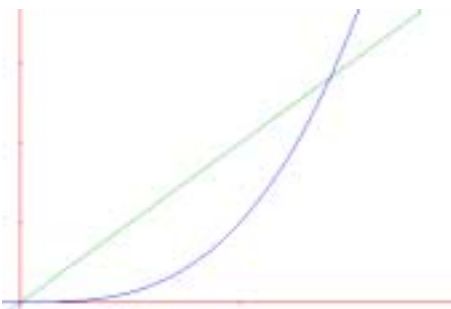


Volumes-by-Cross-Sections Review KEY



The following questions refer to the first-quadrant region bounded by $y = x^3$ and $y = 2x$.

Set-up but do not evaluate the integrals.

Helpful notes for all 10 problems:

$$f(x) = x^3, f(y) = \sqrt[3]{y}, g(x) = 2x, g(y) = \frac{1}{2}y, \text{ functions intersect at } (0,0) \text{ and } (\sqrt{2}, 2\sqrt{2})$$

All of the rotation problems can be done with washers OR shells. Both answers are given and either is acceptable.

1. Find the volume of the region generated when rotated about the x -axis.

$$\text{washers: } \pi \int_0^{\sqrt{2}} (2x)^2 - (x^3)^2 dx, \text{ shells: } 2\pi \int_0^{2\sqrt{2}} y \left(\sqrt[3]{y} - \frac{1}{2}y \right) dy$$

2. Find the volume of the region generated when rotated about the y -axis.

$$\text{washers: } \pi \int_0^{2\sqrt{2}} \left(\sqrt[3]{y} \right)^2 - \left(\frac{1}{2}y \right)^2 dy, \text{ shells: } 2\pi \int_0^{\sqrt{2}} x(2x - x^3) dx$$

3. Find the volume of the region generated when rotated about $y = -2$.

$$\text{washers: } \pi \int_0^{\sqrt{2}} (2x+2)^2 - (x^3+2)^2 dx, \text{ shells: } 2\pi \int_0^{2\sqrt{2}} (y+2) \left(\sqrt[3]{y} - \frac{1}{2}y \right) dy$$

4. Find the volume when cross-sections perpendicular to the x -axis are sides of squares.

$$\int_0^{\sqrt{2}} (2x - x^3)^2 dx$$

5. Find the volume of the region generated when rotated about $x = 2$.

$$\text{washers: } \pi \int_0^{2\sqrt{2}} \left(2 - \frac{1}{2}y \right)^2 - \left(2 - \sqrt[3]{y} \right)^2 dy, \text{ shells: } 2\pi \int_0^{\sqrt{2}} (2-x)(2x - x^3) dx$$

6. Find the volume when cross-sections perpendicular to the y -axis are legs of isosceles right triangles.

$$\int_0^{2\sqrt{2}} \frac{1}{2} \left(\sqrt[3]{y} - \frac{1}{2}y \right)^2 dy$$

7. Find the volume of the region generated when rotated about $y = 5$.

washers: $\pi \int_0^{\sqrt{2}} (5 - x^3)^2 - (5 - 2x)^2 dx$, **shells:** $2\pi \int_0^{2\sqrt{2}} (5 - y) \left(\sqrt[3]{y} - \frac{1}{2}y \right) dy$

8. Find the volume when cross-sections perpendicular to the x -axis are diameters of circles.

$$\int_0^{\sqrt{2}} \pi \left(\frac{2x - x^3}{2} \right)^2 dx$$

9. Find the volume of the region generated when rotated about $x = -3$.

washers: $\pi \int_0^{2\sqrt{2}} \left(\sqrt[3]{y} + 3 \right)^2 - \left(\frac{1}{2}y + 3 \right)^2 dy$, **shells:** $2\pi \int_0^{\sqrt{2}} (x + 3)(2x - x^3) dx$

10. Find the volume when cross-sections perpendicular to the y -axis are sides of squares.

$$\int_0^{2\sqrt{2}} \left(\sqrt[3]{y} - \frac{1}{2}y \right)^2 dy$$