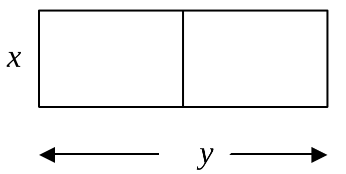
p.372 #1

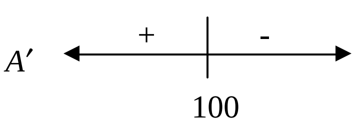


Maximize area:
$A = xy$

Constraint:
$2y + 3x = 600$
$2y = 600 - 3x$
$y = 300 - \frac{3}{2}x$

$A = xy = x\left(300 - \frac{3}{2}x\right)$
$= 300x - \frac{3}{2}x^2$

$A' = 300 - 3x = 0$
$300 = 3x$
$x = 100$

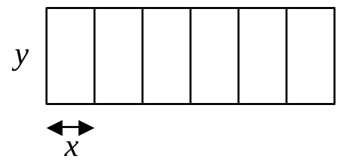


Increases for $x < 100$
Decreases for $x > 100$
$\therefore$ maximum at $x = 100$

$y = 300 - \frac{3}{2}x = 300 - \frac{3}{2}(100)$
$= 150$

Total width: 150 ft
Length: 100 ft

p.372 #2

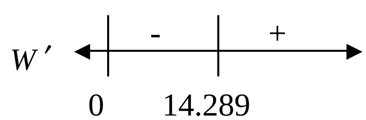


Minimize wall length:
$W = 12x + 7y$

Constraint:
$xy = 350$
$y = \frac{350}{x}$

$W = 12x + 7\left(\frac{350}{x}\right)$
$= 12x + \frac{2450}{x}$

$W' = 12 - \frac{2450}{x^2} = 0$
$12 = \frac{2450}{x^2}$
$x^2 = \frac{2450}{12}$
$x = 14.289$



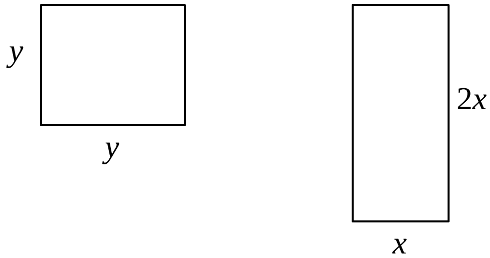
Decreases for $0 < x < 14.289$
Increases for $x > 14.289$
$\therefore$ minimum at $x = 14.289$

$y = \frac{350}{x} = \frac{350}{14.3} = 24.495$

a) Rooms should be 14.289 ft by 24.495 ft

b) 10 rooms: $W = 20x + 11y = 20x + \frac{3850}{x}$
$W' = 20 - \frac{3850}{x^2} = 0$
$x = 13.874$
$y = 25.226$

p.373 #3



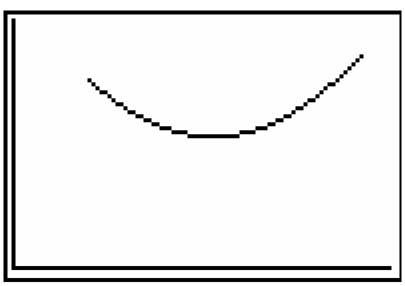
Maximize area:
$A = y^2 + 2x^2$

Constraints:
$4y + 6x = 600$
$y^2 \geq 100$
$2x^2 \geq 800$

a) smallest when $2x^2 = 800$ so $x = 20$
largest when using all fence except minimum used by square (40 ft) so $6x = 560$ $\therefore x = 93.333$
$20 \leq x \leq 93.333$

b)

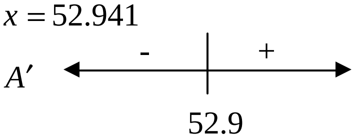
$$4y + 6x = 600 \qquad A = y^2 + 2x^2$$
$$4y = 600 - 6x \qquad = \left(150 - \frac{3}{2}x\right)^2 + 2x^2$$
$$y = 150 - \frac{3}{2}x$$



[0, 100] x [0, 20000]

c)

$$A' = 2\left(150 - \frac{3}{2}x\right)\left(\frac{-3}{2}\right) + 4x = 0$$
$$-450 + \frac{9}{2}x + 4x = 0$$
$$\frac{17}{2}x = 450$$
$$x = 52.941$$



Decreases for $x < 52.941$
Increases for $x > 52.941$
$\therefore$ minimum at $x = 52.941$

Looking for maximum, so consider endpoints:
$A(20) = 15200$
$A(93.3) = 17522.222$

$\therefore$ maximum area is 17,522.222 sq. ft.

p.372 #4

$A = x^2 + pr^2$

Constraints:

$4x + 2pr = 1000$

$r \geq 25$

Maximum circle size

$2pr = 1000$

$r = \frac{500}{p}$

$25 \leq r \leq 159.155$

$4x + 2pr = 1000$

$4x = 1000 - 2pr$

$x = 250 - \frac{p}{2}r$

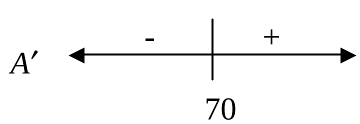
$A = \left(250 - \frac{p}{2}r\right)^2 + pr^2$

$A'' = 2\left(250 - \frac{p}{2}r\right)\left(\frac{-p}{2}\right) + 2pr$

$= -250p + \frac{p^2}{r} + 2pr = 0$

$r\left(\frac{p^2}{2} + 2p\right) = 250p$

$r = 70.012$



Decreases for $r < 70$

Increases for $r > 70$

$\therefore$ minimum at $r = 70$

b) maximum must occur at endpoint

$r(25) = 46370.667$

$r(159.155) = 79577.472$

$\therefore$ maximum area when $r = 159.155$ (all fence used on circle)

p.372 #5

Maximize volume:

$V = x^2 y$

Constraint:

$x^2 + 4xy = 120$

$4xy = 120 - x^2$

$y = \frac{120 - x^2}{4x}$

$V = x^2 y = x^2\left(\frac{120 - x^2}{4x}\right)$

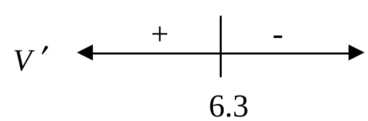
$= 30x - \frac{1}{4}x^3$

$V' = 30 - \frac{3}{4}x^2 = 0$

$30 = \frac{3}{4}x^2$

$40 = x^2$

$x = 6.325$



Increases for $x < 6.3$

Decreases for $x > 6.3$

$\therefore$ maximum at $x = 6.325$

$y = \frac{120 - x^2}{4x} = 3.162$

a) 6.325 cm by 6.325 cm by 3.162 cm

b) conjecture: Depth is half of width

p.372 #7

Minimize cost:

$C = 10(base\ area) + 5(side\ area)$

$= 10(5x) + 5(2xy + 10y)$

Constraint:

$V = 5xy = 72$

$y = \dfrac{72}{5x}$

$C = 50x + 10x\left(\dfrac{72}{5x}\right) + 50\left(\dfrac{72}{5x}\right)$

$= 50x + 144 + \dfrac{720}{x}$

$C' = 50 - \dfrac{720}{x^2} = 0$

$50 = \dfrac{720}{x^2}$

$x^2 = \dfrac{72}{5}$

$x = 3.795$

C'	0	-	3.795	+

Decreases for $0 < x < 3.795$

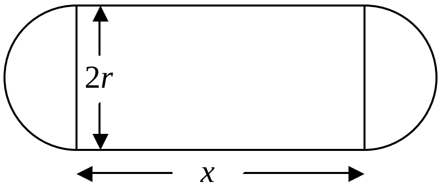
Increases for $x > 3.795$

$\therefore$ minimum at $x = 3.795$

$C(3.795) = 5(3.795) + 144 + \dfrac{720}{3.795}$

$= \$523.47$

p.372 #10



Minimize area:

$A = x(2r) + pr^2$

Constraints:

$r \geq 20$

$x \geq 100$

$P = 400 = 2x + 2pr$

$200 = x + pr$

$x = 200 - pr$

$A = (200 - pr)(2r) + pr^2$

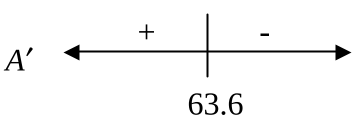
$= 400r - 2pr^2 + pr^2$

$= 400r - pr^2$

$A' = 400 - 2pr = 0$

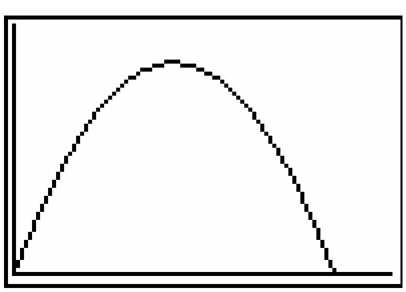
$400 = 2pr$

$r = \frac{200}{p} = 63.662$



Increasing for $x < 63.6$
Decreasing for $x > 63.6$
$\therefore$ maximum at $x = 63.662$

Minimum must occur at endpoint



[0, 150] x [0, 15000]

Smallest *r* is 20.

Largest *r* occurs when least straight lengths are used (i.e. $x = 100$).

$400 = 2(100) + 2pr$

$200 = 2pr$

$r = \frac{100}{p}$

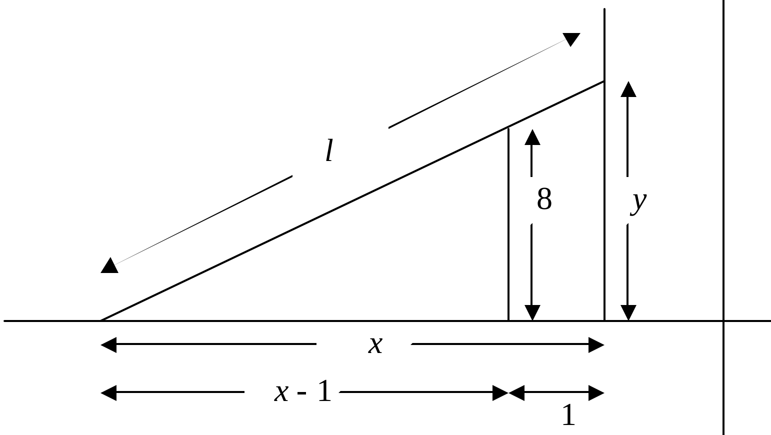
$A(20) = 6743.363$

$A\left(\frac{100}{p}\right) = 9549.297$

$\therefore$ minimum occurs at $r = 20$

$x = 200 - pr = 137.168$

p.372 #11



Minimize length of ladder, *l*:
$l^2 = x^2 + y^2$

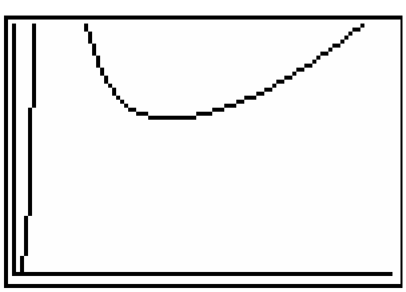
Relate *x* and *y* using similar triangles:
$\frac{y}{x} = \frac{8}{x-1} \Rightarrow y = \frac{8x}{x-1}$

Re-write *l* in one variable:
$l^2 = x^2 + \left(\frac{8x}{x-1}\right)^2 = x^2 + \frac{64x^2}{(x-1)^2}$

Find minimum using derivative:
(minimum l^2 is equivalent to minimum *l*)
$(l^2)' = \frac{(x-1)^2(128x) - 64x^2(2(x-1))}{(x-1)^4}$

YUCK! TIME TO USE MY CALCULATOR!

Graph l^2:

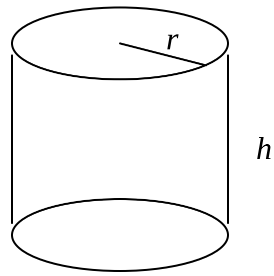


[0, 12] x [0, 200]

Minimum occurs at (5, 125)

Since minimum l^2 is 125, minimum *l* is approximately 11.180 feet.

p.372 #13

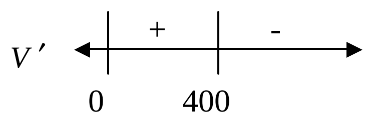


Maximize volume of cylinder:
$V = pr^2h$

Relate *r* and *h* using given perimeter:
$1200 = 2r + 2h$
$600 = r + h$
$h = 600 - r$

Re-write *V* in one variable:
$V = pr^2(600 - r) = p(600r^2 - r^3)$

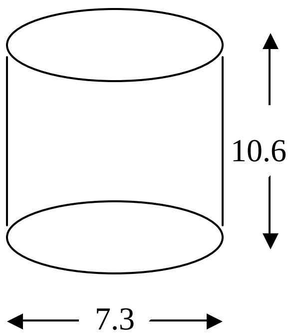
Find maximum using derivative:
$V' = p(1200r - 3r^2) = 0$
$3r(400 - r) = 0$
$r = 0, 400$



Max occurs when $r = 400$, so $h = 600 - 400 = 200$

400 mm radius and 200 mm height

p.372 #15



a)

$$V = pr^2h$$
$$= p(3.65)^2(10.6)$$
$$= 443.651$$

b)

$$A = 2pr^2 + 2prh$$

Relate r and h:

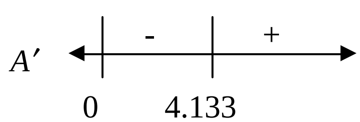
$$pr^2h = 443.651$$
$$h = \frac{443.651}{pr^2}$$

Rewrite A in terms of r:

$$A = 2pr^2 + 2pr\left(\frac{443.651}{pr^2}\right)$$
$$= 2pr^2 + \frac{887.302}{r}$$

c) Find minimum using derivative:

$$A' = 4pr - \frac{887.302}{r^2} = 0$$
$$4pr = \frac{887.302}{r^2}$$
$$r^3 = 70.609$$
$$r = 4.133$$



$$h = \frac{443.651}{p(4.133)^2} = 8.266$$

4.133 cm radius, 8.266 cm height
Can is short and fat
Ratio of diameter to altitude is 1 to 1

d) $A_{normal} = 2p(3.65)^2 + 2p(3.65)(10.6) = 326.804\ cm^2$

$A_{minimum} = 2p(4.133)^2 + 2p(4.133)(8.266) = 322.015\ cm^2$

The normal can uses close to the same amount of metal

$$\frac{326.804 - 322.015}{326.804} = 0.0147$$

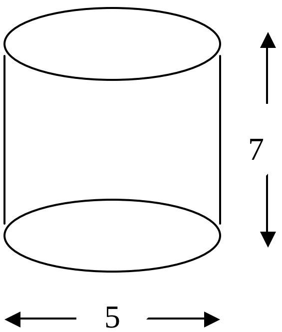
Normal can uses about 1.5% more metal

e)

$$\left(\frac{20000000\ cans}{day}\right)\left(\frac{365.2522\ days}{year}\right)\left(\frac{\$0.06}{can}\right)(1.47\%)$$
$$= \$6,432,715.24$$

about $6.4 million

p.372 #17



a)

$V = pr^2 h = p(2.5)^2(7)$
$= 137.445$

$A = pr^2 + 2prh$

Relate r and h together:

$pr^2 h = 137.445$

$h = \dfrac{137.445}{pr^2}$

Re-write A in terms of one variable:

$$A = pr^2 + 2pr\left(\frac{137.445}{pr^2}\right) = pr^2 + \frac{274.889}{r}$$

Minimize A:

$A' = 2pr - \dfrac{274.889}{r^2} = 0$

$r^3 = 43.75$

$r = 3.524$

$$h = \frac{137.445}{p(3.524)^2} = 3.524$$

Dimensions: radius 3.524 cm, height 3.524 cm

b) Ratio of diameter to altitude is 2 to 1

c)

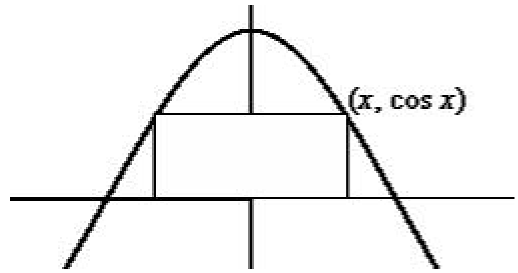
$$\left(\frac{\$2.00}{m^2}\right)\left(\frac{300000000\ cups}{year}\right)\left(\frac{129.591\ cm^2}{cup}\right)\left(\frac{1\,m}{100\ cm}\right)\left(\frac{1\,m}{100\ cm}\right) = \frac{\$7{,}775{,}441.82}{year}$$

$$\left(\frac{\$2.00}{m^2}\right)\left(\frac{300000000\ cups}{year}\right)\left(\frac{117.019\ cm^2}{cup}\right)\left(\frac{1\,m}{100\ cm}\right)\left(\frac{1\,m}{100\ cm}\right) = \frac{\$7{,}021{,}141.88}{year}$$

Savings per year: $754,299.93

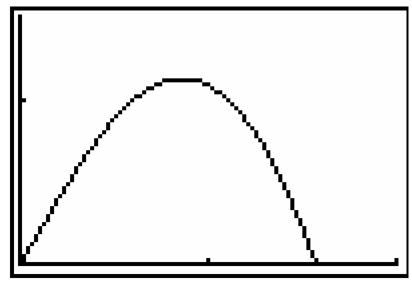
d) left to you ☺

p.372 #19



$A = 2x(\cos x)$

Find maximum by graphing A:



[0, 2] x [0, 1.5]

Maximum at (0.860, 1.122)

$x = 0.860$, $A = 1.122$